Product rule, three or more functions

As we learned in the last section, product rule is a derivative rule that allows us to take the derivative of a function which is itself the product of two other functions. Product rule tells us that the derivative of an equation like

$$y = f(x)g(x)$$

will look like this:

$$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$$

But what do we do if our function is the product of more than two functions? In this situation, if our function is

$$y = f(x)g(x)h(x)$$

then the derivative looks like this:

$$\frac{dy}{dx} = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

We can see that the original function was a product of three functions, and its derivative was the sum of three products. If our function was the product of four functions, the derivative would be the sum of four products.

As you can see, when we take the derivative using product rule, we take the derivative of one function at a time, multiplying by the other two original functions. To be more specific, we take the derivative of $f(x)$, and

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multiply it by $g(x)$ and $h(x)$, leaving those two as they are. Then we add to that the derivative of $g(x)$, multiplied by $f(x)$ and $h(x)$ left as they are. We can continue this pattern, taking the derivative of only one of the functions and leaving the others alone, for as many functions as are multiplied together in our original problem.

Looking at an equation which is the product of four functions, like

$$y = f(x)g(x)h(x)j(x)$$

the derivative is

$$\frac{dy}{dx} = f'(x)g(x)h(x)j(x) + f(x)g'(x)h(x)j(x) + f(x)g(x)h'(x)j(x) + f(x)g(x)h(x)j'(x)$$

Example

Use product rule to find the derivative.

$$y = \left(4x^6\right)\left(\sin 3x\right)\left(-6e^{2x}\right)$$

Applying product rule, we get

$$\frac{dy}{dx} = \left(24x^5\right)\left(\sin 3x\right)\left(-6e^{2x}\right) + \left(4x^6\right)\left(3\cos 3x\right)\left(-6e^{2x}\right) + \left(4x^6\right)\left(\sin 3x\right)\left(-12e^{2x}\right)$$

$$\frac{dy}{dx} = -144x^5e^{2x}\sin 3x - 72x^6e^{2x}\cos 3x - 48x^6e^{2x}\sin 3x$$

$$\frac{dy}{dx} = -24x^5e^{2x}(6\sin 3x + 3x\cos 3x + 2x\sin 3x)$$

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Let's do another example with more functions.

Example

Use product rule to find the derivative.

$$y = \left(8x^{12}\right)\left(\frac{6x^2}{7}\right)\left(-2e^x\right)\left(-6\cos 5x\right)\left(\sin 4x\right)$$

Applying product rule, we get

$$\frac{dy}{dx} = \left(96x^{11}\right)\left(\frac{6x^2}{7}\right)\left(-2e^x\right)\left(-6\cos 5x\right)\left(\sin 4x\right)$$

$$+\left(8x^{12}\right)\left(\frac{12x}{7}\right)\left(-2e^x\right)\left(-6\cos 5x\right)\left(\sin 4x\right)$$

$$+\left(8x^{12}\right)\left(\frac{6x^2}{7}\right)\left(-2e^x\right)\left(-6\cos 5x\right)\left(\sin 4x\right)$$

$$+\left(8x^{12}\right)\left(\frac{6x^2}{7}\right)\left(-2e^x\right)\left(30\sin 5x\right)\left(\sin 4x\right)$$

$$+\left(8x^{12}\right)\left(\frac{6x^2}{7}\right)\left(-2e^x\right)\left(-6\cos 5x\right)\left(4\cos 4x\right)$$

Pulling out common factors, we get

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$$\frac{dy}{dx} = \left[(8x^{11})\left(\frac{6x}{7}\right)(-2e^x)(-6)\right] \cdot \Big[(12)(x)(\cos 5x)(\sin 4x)$$

$$+(x)(2)(\cos 5x)(\sin 4x)$$

$$+(x)(x)(\cos 5x)(\sin 4x)$$

$$+(x)(x)(-5\sin 5x)(\sin 4x)$$

$$+(x)(x)(\cos 5x)(4\cos 4x)\Big]$$

And simplifying, we get

$$\frac{dy}{dx} = \left(\frac{576}{7}e^x x^{12}\right) \cdot \Big[12x\sin 4x\cos 5x$$

$$+2x\sin 4x\cos 5x$$

$$+x^2\sin 4x\cos 5x$$

$$-5x^2\sin 4x\sin 5x$$

$$+4x^2\cos 4x\cos 5x\Big]$$

$$\frac{dy}{dx} = \left(\frac{576}{7}e^x x^{13}\right) \cdot \Big[12\sin 4x\cos 5x$$

$$+2\sin 4x\cos 5x$$

$$+x\sin 4x\cos 5x$$

$$-5x\sin 4x\sin 5x$$

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$$+4x\cos 4x\cos 5x\Big]$$

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